# Elliptic Curves and Mordell's Theorem

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In this essay I shall provide an introduction to Elliptic Curves, and how the consideration of rational points on a non-singular elliptic curve form a finitely generated group. This result is proven in Mordell's theorem. I will then discuss the consequences of Mordell's theorem, such as the implication that the rank of an elliptic curve is always finite, despite there being no effective method for calculation of the rank of a given elliptic curve.
The rank of an elliptic curve also forms the basis of the Birch and Swinnerton-Dyer Conjecture, which relates the rank of an elliptic curve to the order of the associated L-function.

An elliptic curve can be represented as a cubic polynomial in two variables, and by considering geometric constructions in the projective plane, a group operation between the rational points on an elliptic curve can be introduced.
Mordell's theorem states that if a non-singular plane cubic curve has a rational point, then the group of rational points is finitely generated. Proved by Louis Mordell in 1922, this answers a question conjectured by Poincaré at the start of the 20th century.